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TRANSVERSE VIBRATIONS OF A SIMPLY SUPPORTED RECTANGULAR ORTHOTROPIC PLATE WITH A CIRCULAR PERFORATION WITH A FREE EDGE

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1. introduction

The structural designer is frequently confronted with the need to provide convenient passage for utility ducts through openings in slabs and beams. The situation leads to a more complicated response from a structural dynamics viewpoint.

A very thorough study has been performed by Abdalla and Kennedy in the case of pre-stressed concrete beams [1]. The determination of the fundamental frequency of transverse vibration of simply supported isotropic and orthotropic rectangular plates with rectangular holes has been tackled by the authors using analytical and numerical techniques [2–3].

The present study deals with the determination of the lower natural frequencies of transverse vibration of simply supported orthotropic plates with circular openings (Figure 1) using two independent techniques.

(1) An analytical approach whereby the displacement amplitude is expressed in terms of a truncated double Fourier series which is the exact solution of the vibrating, solid rectangular plate simply supported at its four edges. The frequency determinant is generated using the Rayleigh–Ritz method.

(2) The finite element method, using a standard code [4].

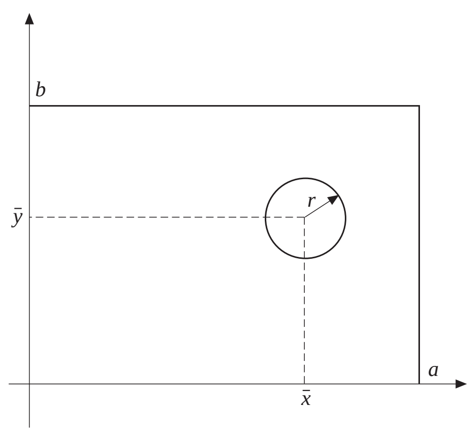

Figure 1. Simply supported orthotropic rectangular plate with a circular cutout.

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2. ANALYTICAL SOLUTION

Following previous studies [2, 3] and using Lekhnitskii's well established notation [5] one expresses the governing functional in the form

$$J[W]=\frac{1}{2}\int\int\left[D_1\left(\frac{\partial^2 W}{\partial x^2}\right)^2+2D_1\nu_2\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2}+D_2\left(\frac{\partial^2 W}{\partial y^2}\right)^2\right.$$

$$\left.+4D_k\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right]\mathrm{d}x\,\mathrm{d}y-\frac{\rho\omega^2}{2}\int\int hW^2\,\mathrm{d}x\,\mathrm{d}y. \tag{1}$$

The displacement amplitude is now approximated using the truncated expression

$$W(x,y)\simeq W_\alpha(x,y)=\sum_{n=1}^{N}\sum_{m=1}^{M}A_{nm}\sin(n\pi x/a)\sin(m\pi y/b), \tag{2}$$

where each co-ordinate function $\sin(n\pi y/a)\sin(m\pi y/b)$ constitutes the exact eigenfunction when the orthotropic, simply supported, rectangular plate is simply connected and, clearly, does not satisfy the governing, natural boundary conditions at the cutout. However, the procedure is a legitimate one when employing the Rayleigh–Ritz method.

TABLE 1

Frequency coefficients Ω_i ($i = 1, 2, 3, \ldots, 5$) *determined in the case of a simply supported rectangular orthotropic plate* ($D_2/D_1 = D_k/D_1 = 0{\cdot}5$; $\nu_2 = 0{\cdot}3$) *with a circular hole with a free edge* ($b/a = 2/3$)

r/a	$\bar{x}/a$	$\bar{y}/b$	Method	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
			Analytical and F.E.	30·23	63·92	79·52	115·19	120·95
0·10	1/4	1/4	F.E.	29·75	64·17	79·37	112·89	127·53
			400 terms	28·95	63·91	73·38	104·42	
	1/4	1/2	F.E.	30·27	65·72	77·40	113·91	118·65
			400 terms	30·45	65·58	78·06	117·01	
			900 terms	30·23	65·53	77·13	114·05	
	1/2	1/2	F.E.	30·76	62·32	77·53	115·54	118·05
			400 terms	30·66	64·12	78·66	115·84	
			900 terms	30·76	61·70	77·89	115·61	
0·15	1/4	1/4	F.E.	29·22	64·51	79·21	110·83	138·15
			400 terms	28·99	64·29	78·50	110·45	
	1/4	1/2	F.E.	30·48	70·98	75·70	114·52	119·09
			400 terms	30·31	71·24	74·60	113·35	
	1/2	1/2	F.E.	32·85	59·04	75·37	111·94	123·59
			400 terms	33·07	57·55	74·24	111·11	
0·20	1/4	1/4	—	—	—	—	—	—
	1/4	1/2	F.E.	30·64	74·88	80·30	123·41	126·48
			400 terms	30·26	70·40	81·18	117·90	
	1/2	1/2	F.E.	37·07	56·63	77·16	109·16	130·35
			400 terms	36·73	54·63	74·62	108·27	

Obviously, the energy functional is integrated over the physical doubly connected domain in expression (1).

Substituting the approximating function in equation (1) and minimizing $J(W)$ with respect to the A_{nm}'s results in an homogeneous, linear system of equations in the A_{nm}'s. The non-triviality condition yields a secular determinant the roots of which constitute the frequency coefficients $\Omega_i = \sqrt{\rho h/D_1}\omega_i a^2$.

3. NUMERICAL RESULTS

The first five frequency coefficients $\Omega_i = \sqrt{\rho h/D_1}\omega_i a^2$ have been determined for a simply supported orthotropic plate with a circular perforation for $D_2/D_1 = D_k/D_1 = 0{\cdot}5$ and $\nu_2 = 0{\cdot}3$, for different sizes and positions of the circular hole.

Values of Ω_i for $b/a = 2/3$, 1 and 3/2, are depicted in Tables 1, 2 and 3, respectively. The value of Ω_5 was only determined by means of the finite element method, since the analytical approach did not converge properly for the fifth eigenvalue.

For $b/a = 2/3$, a net of 67×100 elements was used for the solid case; for $b/a = 1$, 100×100 elements, and for $b/a = 3/2$, 120×80 elements. The number of degrees of freedom varied as a function of the type of configuration; for instance, for $a/b = 1$, $r/a = 0{\cdot}1$, 28 920 degrees of freedom were used independently for the location of the hole. The discretization of the plate in the vicinity of the orifice is depicted in Figure 2.

TABLE 2

Frequency coefficients Ω_i $(i = 1, 2, 3, \ldots, 5)$ *determined in the case of a simply supported rectangular orthotropic plate* $(D_2/D_1 = D_k/D_1 = 0{\cdot}5;\ \nu_2 = 0{\cdot}3)$ *with a circular hole with a free edge* $(b/a = 1)$

r/a	$\bar{x}/a$	$\bar{y}/b$	Method	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
			Analytical and F.E.	19·98	43·47	51·19	79·52	79·95
0·10	1/4	1/4	F.E.	19·75	43·46	51·08	78·47	82·46
			400 terms	19·69	43·31	50·85	78·41	
	1/4	1/2	F.E.	19·95	42·57	51·28	78·97	79·41
			400 terms	19·72	42·47	51·59	77·79	
	1/2	1/2	F.E.	20·09	43·04	50·24	77·14	80·39
			400 terms	20·55	42·80	47·27	77·22	
0·15	1/4	1/4	F.E.	19·51	43·33	51·57	77·18	86·95
			400 terms	19·48	43·14	51·63	77·13	
	1/4	1/2	F.E.	20·05	41·54	53·05	77·47	79·06
			400 terms	18·71	41·88	52·85		
	1/2	1/2	F.E.	20·79	42·16	48·18	75·13	81·79
			400 terms	20·72	41·57	43·25	73·75	
0·20	1/4	1/4	F.E.	19·20	42·67	52·50	76·26	90·56
			400 terms	19·31	42·22	53·13	75·86	
	1/4	1/2	F.E.	20·25	40·50	55·87	76·40	79·17
			400 terms	19·53	39·72	53·34	73·68	
	1/2	1/2	F.E.	22·35	41·41	46·09	73·34	81·94
			400 terms	22·26	38·96	49·65	73·12	

TABLE 3

Frequency coefficients Ω_i ($i = 1, 2, 3, \ldots, 5$) determined in the case of a simply supported rectangular orthotropic plate ($D_2/D_1 = D_k/D_1 = 0{\cdot}5$; $\nu_2 = 0{\cdot}3$) with a circular hole with a free edge ($b/a = 3/2$)

r/a	$\bar{x}/a$	$\bar{y}/b$	Method	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5
			Analytical and F.E.	14·82	26·49	43·48	44·93	59·28
0·10	1/4	1/4	F.E.	14·69	26·50	43·15	44·71	59·80
			20 terms	14·69	26·49	43·24	44·73	
	1/4	1/2	F.E.	14·73	26·09	43·40	44·62	58·91
			20 terms	14·71	26·12	43·39	44·52	
	1/2	1/2	F.E.	14·72	26·40	43·85	44·41	58·12
			20 terms	14·68	26·42	43·75	44·36	
0·15	1/4	1/4	F.E.	14·55	26·60	42·55	45·12	61·62
			20 terms	14·54	26·58	42·53	45·16	
	1/4	1/2	F.E.	14·69	25·60	42·91	45·95	57·97
			20 terms	14·56	25·41	42·69	45·57	
	1/2	1/2	F.E.	14·87	26·09	42·83	45·22	57·01
			20 terms	14·67	25·75	41·86	44·87	
0·20	1/4	1/4	F.E.	14·38	26·73	41·63	45·82	63·95
			20 terms	14·33	26·61	41·62	45·84	
	1/4	1/2	F.E.	14·70	24·93	41·97	48·51	56·62
			20 terms	14·55	25·00	41·52	48·45	
	1/2	1/2	F.E.	15·33	25·61	41·09	47·47	55·99
			20 terms	16·13	25·92	38·88	49·58	
			30 terms	15·14	24·80	39·34	47·42	

The analytical determinations were performed making $N = M = 20$. Accordingly a (400 × 400) secular determinant was generated. Numerical experiments were performed

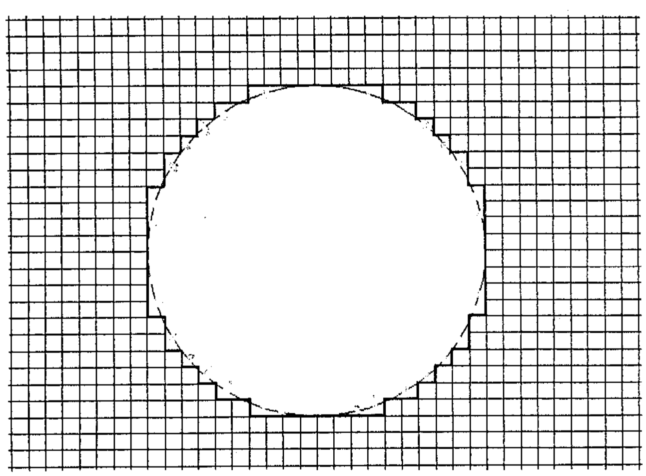

Figure 2. Modelling of the orthotropic plate in the neighborhood of the orifice, using square elements ($a/b = 1$, $r/a = 0{\cdot}1$).

taking $N = M = 10$ and a good rate of convergence was observed when increasing N and M, up to $N = M = 20$.

On the other hand, numerical instabilities were observed, in general, when the frequency determinant was enlarged to 900×900; see, for instance, Table 1, $r/a = 0{\cdot}10$, $\bar{x}/a = 1/4$, $\bar{y}/b = 1/2$ and $\bar{x}/a = 1/2 = \bar{y}/b$ and Table 3, $r/a = 0{\cdot}20$, $\bar{x}/a = 1/2 = \bar{y}/b$. On the other hand, the agreement between analytical and finite element predictions is very good from a practical viewpoint.

From a dynamic stiffening viewpiont, one observes an appreciable increment in the fundamental frequency, with respect to the solid plate, for $r/a = 0{\cdot}2$ and $\bar{x}/a = 1/2 = \bar{y}/b$ in the case of Table 1 ($b/a = 2/3$) and for $b/a = 1$ (Table 2), where increments of the order of 20% and 10%, respectively, are observed.

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